2025
厦大杯中学物理比赛
XMUM Cup Physics Competition for Secondary School

Question	Acceptable answers		Question	Acceptable answers
1	25		16	59
2	45		17	1.3
3	14		18	-2.6
4	4.9		19	52
5	0.94		20	1.2
6	1.8		21	0.28
7	0.19		22	1.5
8	17		23	0.28
9	0.56		24	39
10	0.62		25	1.0
11	7.9		26	67
12	92		27	-390
13	87		28	2.4
14	25		29	-1.6,0.85 -1.60,0.85 -1.65,0.85 -1.55,0.85
15	12		30	1.6

Topics	Questions	No. Questions
Mechanics	1,2,3,4,5,6,7,8,9,10	10
Thermal Physics	13,14,15,30	4
Electromagnetism	21,22,23,24	4
Electronics Circuit	18,19,20	3
Gravitation	11,25	2
Modern Physics	26,27,28	3
Waves	12,17	2
Optics	1	1
Measurement	29	1
	Total	30

所有答案必定是 2 位有效数字的数, 除非题目另有要求。不要把步或和单位填入空格中。

All answers must be number with TWO significant figures unless otherwise stated in the question. DO NOT include steps or units in the answer box.

Questions 1

A car is accelerating uniformly from rest. After 5 s, the velocity of the car reaches 10 m/s. Find the distance in **meters (m)** the car travels in 5 s.

某车自静止开始匀加速。5 s 后，车速达到了 10 m/s。求此车在这 5 s 内行驶的距离，以 **meters (m)**为单位。

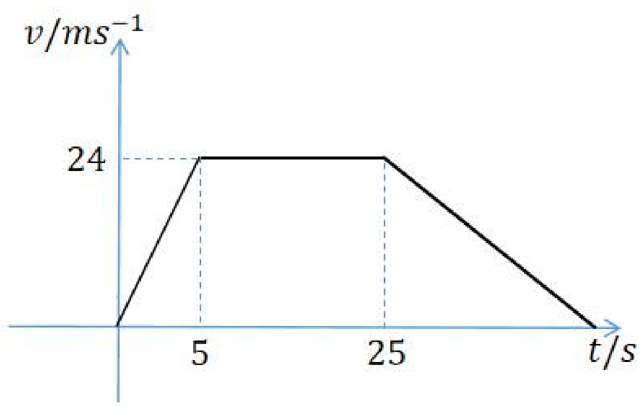
Ans: 25

$$a=10/5=2\ m/s^2$$
$$v^2-v_0^2=2as \Rightarrow s=10^2/(2*2)=25m$$

Questions 2

The figure below shows the velocity of a car versus time. The car accelerates uniformly from rest for 5 s and then moves with a constant velocity of 24 m/s until 25 s. After that, the car decelerates uniformly and stops at *t* seconds. If the total distance traveled by the car in *t* seconds is 780 m, find the value of *t*.

下图表示某车的速度与时间的关系图。此车由静止开始匀加速了 5 s 后就维持 24 m/s 的匀速度前进直至 25 s。 然后此车开始作匀减速运动并在 *t* 秒时达到静止。若此车在 *t* 秒内行驶的距离为 780 m，求 *t* 的值。



Ans: 45

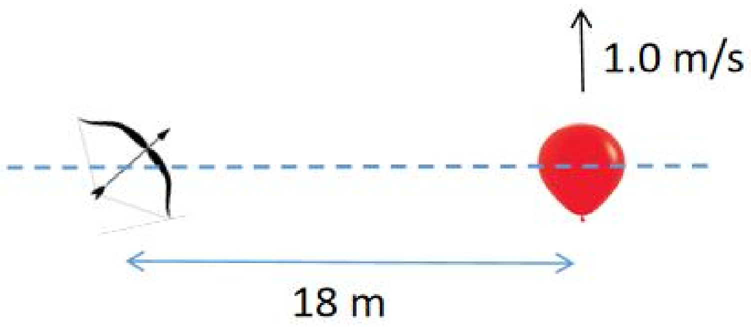
The area equals to the distance travelled,

$$s=24(20+t)/2=780$$
$$t=45$$

Question 3

Initially, both the balloon and the arrow are at the same level, 18 m apart. When the balloon is released, the arrow is shot at the same time at an angle of 45° to the horizontal. If the balloon travels upward with a constant velocity of 1.0 m/s, what should be the initial speed of the arrow (in m/s) in order to hit the balloon successfully? Assume air resistance is negligible.

开始时，气球与弓箭都处在同一水平线上，相距 18 m。 当气球被释放后，弓箭立刻以仰角45°射出。若气球以 1.0 m/s 匀速上升， 弓箭的初始速率必须为多少 m/s 方能成功击中气球？(空气阻力可以忽略不计。)



Ans: 14

Both arrow and balloon are at the same height

$$v_b t = (v\sin\theta)t - \frac{1}{2}gt^2 \Rightarrow v_b = v\sin\theta - \frac{1}{2}gt$$

Also, both the arrow and the balloon need to be at the same horizontal position

$$(v\cos\theta)t = 18$$

This leads to

$$-2v_b + 2v\sin\theta - 18g/(v\cos\theta) = 0 \Rightarrow v^2 - \sqrt{2}v_b v - 18g = 0$$

Solving the above equation, we obtain

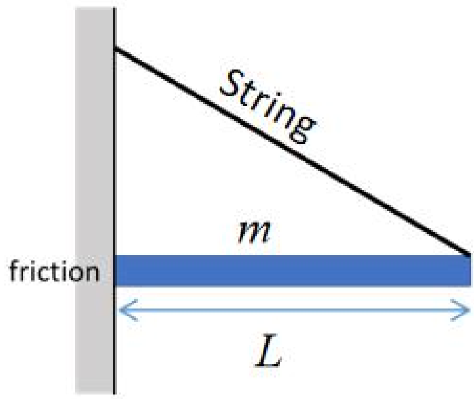
$$v = \frac{1}{2}\left(\sqrt{2} \pm \sqrt{2 + 4\times 18g}\right)$$

Take the positive solution, we find $v = 14.0\ m/s$.

Questions 4

A uniform rod of mass m=1.0 kg and length L=1.0 m is horizontally suspended from a wall by a thin string, as shown in the figure. One end of the rod is pressed tightly against the wall so that the whole system remains in equilibrium. Find the magnitude of the friction (in Newtons, N) between the wall and the rod.

一长度 L=1.0 m，质量 m=1.0 kg 的均匀木棒被一细弦水平吊挂于墙上，如下图所示。木棒的一端紧贴在墙面使整个系统维持平衡。求墙和木棒之间摩擦力的大小，以 Newtons (N) 为单位。



Ans: 4.9

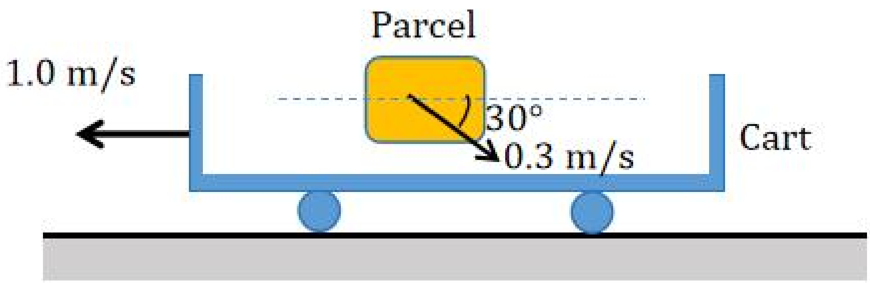
Calculate the torque by using the string and the rod connecting point as the pivot. This gives

$$-fL+\frac{L}{2}mg=0\Rightarrow f=\frac{1}{2}mg=4.9\ N$$

Question 5

When a cart is moving towards the left at a velocity of $1.0\ \mathrm{m/s}$, a parcel with mass 1.0 kg drops into the cart at a velocity $0.3\ \mathrm{m/s}$ in the direction of $30°$ below the horizontal. The parcel moves to left together with the cart at the same velocity v after collision. Given that the mass of the cart is 20 kg, determine the magnitude of v in m/s.

当一滑车正以 $1.0\ \mathrm{m/s}$ 速度向左滑动时，一个 1.0 kg 的包裹以 $0.3\ \mathrm{m/s}$，$30°$俯角的速度落入滑车中。包裹与滑车碰撞后，与滑车以相同速度 v 继续向左移动。已知滑车的质量为 20 kg，求 v 的大小，以 m/s 为单位。



Ans: 0.94

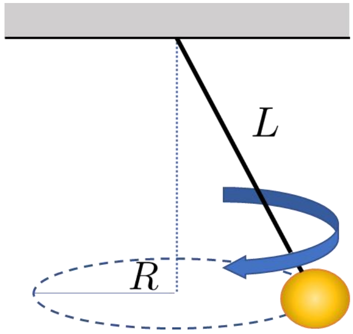
The momentum is not conserved in the vertical direction. Only the horizontal component is relevant.

$$Mv_c - mv_p \cos\theta = (M+m)v \Rightarrow v = \frac{Mv_c - mv_p \cos\theta}{M+m} = 0.94\ m/s$$

Question 6

A conical pendulum of length L swings in a horizontal circle of radius R. If L=1.0 m and the R=0.6L, determine the period (in seconds) of this circular motion.

一摆长为 L 的圆锥摆在水平面内以半径为 R 的圆摆动。若 L=1.0 m，且 R=0.6L，求圆周运动的周期，以秒为单位。



Ans: 1.8

The centripetal force gives

$$mg\tan\theta=\frac{mv^2}{R}\Rightarrow v=\sqrt{gR\tan\theta}$$

The period is

$$T=\frac{2\pi R}{v}=2\pi\sqrt{\frac{R}{g\tan\theta}}=2\pi\sqrt{\frac{L}{g}\sqrt{1-R^2/L^2}}=2\pi\sqrt{\frac{\sqrt{3}}{2g}}=1.795\ s$$

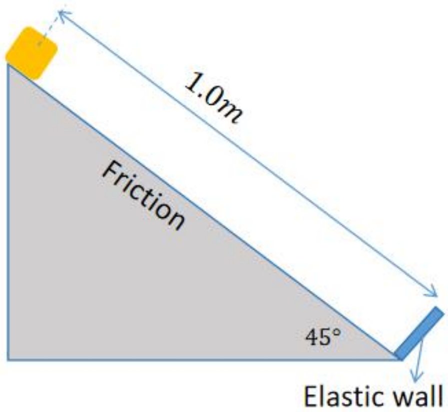
where

$$\tan\theta=\frac{R}{\sqrt{L^2-R^2}}$$

Question 7

A wooden block of mass 0.5 kg is initially placed at the top of an incline plane with an angle 45°. At the bottom of the plane, 1.0 m away from the block, there is an elastic wall. The block starts sliding down the plane from rest, collides elastically with the wall, and then slides back up along the plane. When the block's speed decreases to zero, it is still 32 cm away from its initial position. Find the coefficient of kinetic friction between the plane and the block.

质量为 0.5 kg 的木块一开始被置放在倾角为45°的斜面顶部。在斜面底部，离木块 1.0 m 处有一弹性墙。木块由静止开始沿斜面下滑，与弹性墙作弹性碰撞后，又沿着斜面上滑。当木块速度降为零时，其离开始位置还有 32 cm。求木块与斜面之间的动摩擦系数。



Ans: 0.19

Treat the friction $f = \mu_k mg\cos\theta$ as the external force. Therefore, the change of total mechanics energy equal to the work done by the external force

$$-fx - f(x-\Delta x) = mg(x-\Delta x)\sin\theta - mgx\sin\theta$$

This leads to

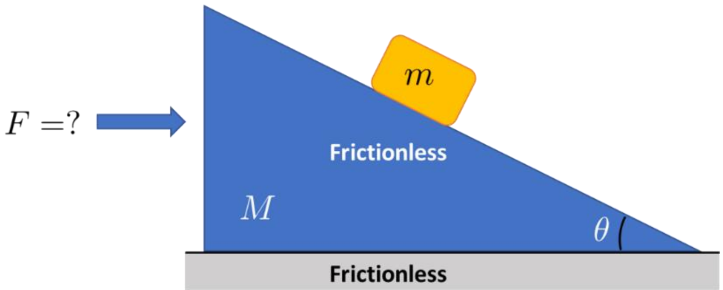
$$\mu_k(2x-\Delta x) = \Delta x\tan\theta \Rightarrow \mu_k = \frac{\Delta x\tan\theta}{(2x-\Delta x)}$$

$$\frac{0.32\tan 45°}{(2.0-0.32)} = 0.19$$

Question 8

A wedge of mass M=2.5 kg is placed on a smooth horizontal surface. The inclined plane of the wedge is also smooth and makes an angle of $\theta=30°$ with the horizontal. A metal block of mass m=0.5 kg is placed on the inclined plane, and a horizontal force F is applied to the wedge so that the metal block remains stationary relative to the inclined plane. Find the minimum force F in Newtons (N).

质量为 M=2.5 kg 的楔块被置放于光滑的水平面上。楔块的斜面也是光滑的，且与水平面成 $\theta=30°$角。在楔块的斜面上放了另一块质量为 m=0.5 kg 的金属块后，以水平力 F 推动楔块以使金属块相对于斜面静止。求 F 的最小值，以 Newtons (N)为单位。



Ans: 17

The acceleration of the entire system

$$F=(M+m)a$$

The net force on the small block on the wedge

$$N\sin\theta=ma$$

$$N\cos\theta-mg=0$$

This gives

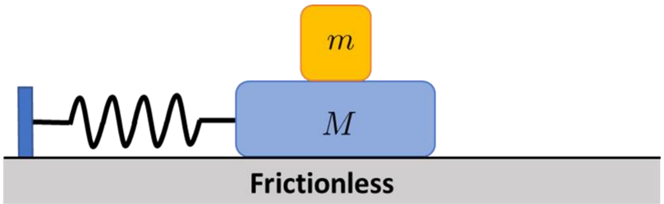
$$F=(M+m)g\tan\theta$$

$$F=(2.5+0.5)\ 9.8\tan 30°=16.97\ N$$

Question 9

On a smooth horizontal surface, a wooden block of mass M=2.0 kg is attached to a spring with a spring constant 14 N/m. The other end of the spring is fixed to a wall. Another wooden block of mass m=0.5 kg is placed on top of block M. If the coefficient of static friction between the two blocks is μ_s=0.32, determine the maximum amplitude of oscillation (in meters) such that no relative sliding occurs between the two blocks.

光滑的水平面上，一质量为 M=2.0 kg 的木块与弹簧系数为 14 N/m 的弹簧相联。弹簧的另一端固定在墙上。另一质量为 m=0.5 kg 的木块被放在 M 的上方。若两木块间的静摩擦系数为μ_s=0.32，当弹簧振动时，求两木块不发生相对滑动的最大振幅。以 meters(m)为单位。



Ans: 0.56

The maximum force of the system is

$$(M+m)a=kA$$

where a is the maximum acceleration and A is the amplitude. In order to hold the small block, the static friction cannot produce the acceleration larger than a. This gives the maximum friction as

$$f=\mu_s mg=ma$$

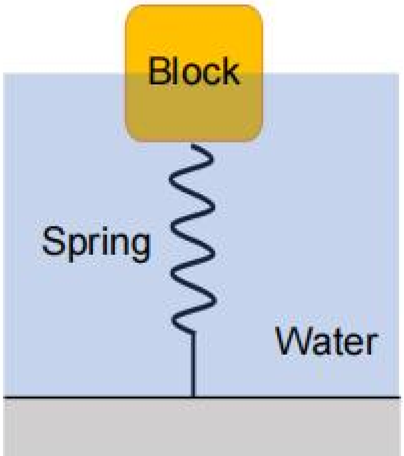
$$A=\frac{(M+m)\mu_s g}{k}$$

$$A=\frac{(2.0+0.5)(0.32)(9.8)}{14}=0.56\ m$$

Question 10

A cubic block with mass 6.25 kg and side 0.2 m is partially submerged in water and attached to a vertical, light spring with spring constant 250 N/m. The other end of the spring is fixed to the bottom. The system is initially in equilibrium, with the spring is stretched and balanced the buoyant force. Given that the density of water is 1000 kg/m^3, calculate the period (in seconds) of oscillation when the block is displaced slightly from equilibrium.

一质量为 6.25 kg, 边长为 0.2 m 的方块部分浸没在水中，并与一弹簧系数为 250 N/m 的轻弹簧竖直连接。弹簧的另一端被固定在水底。系统最初处于平衡状态，此时弹簧被拉伸以平衡浮力。已知水的密度为 1000 kg/m^3，当木块稍微偏离平衡位置时，求其振动的周期，以秒为单位。



Ans: 0.62

At equilibrium, the net force for the system is zero. These yields

$$-k\Delta x_0+\rho(Ad)g-mg=0$$

where Δx_0 is the extension of the spring at equilibrium and d is the depth of the block that immerse in water at equilibrium. Here, $A=L^2$ is the cross section of the cube. As the system deviate the equilibrium position by x. The net force of the block become

$$F=-k(x+\Delta x_0)+\rho A(-x+d)g-mg$$

Using the equilibrium condition, we find

$$F=-kx-\rho Axg$$

The angular frequency is

$$\omega=\sqrt{\frac{k+\rho Ag}{m}}=\sqrt{\frac{250+1000(0.04)9.8}{6.25}}=\sqrt{102.72}\ s$$

The period

$$T=2\pi/\sqrt{102.72}=0.62\ s$$

Question 11

Two identical stars with mass 2.0×10^{30} kg rotating in circular motion with respect to each other at constant speed. If the distance between the two stars is 5.0 AU (1 AU=1.5×10^{11} m, distance between sun and earth), what is the rotation period (in years) of this binary star system? Given that G=6.7×10^{-11} $m^3kg^{-1}s^{-2}$, 1 year has 365 days, 1 day = 24 hours.

两颗一模一样的星体相互绕着对方以等速率作圆周运动。两星体个别的质量为 2.0×10^{30} kg，相距 5.0 AU (1 AU=1.5×10^{11} m, 太阳与地球的距离。). 求此双星系统的运动周期，以年为单位。(取 G=6.7×10^{-11} $m^3kg^{-1}s^{-2}$，1 年=365 天，1 天=24 时)

Ans: 7.9

The forces experience for each

$$\frac{Gm^2}{d^2}=\frac{mv^2}{r}\Rightarrow v=\sqrt{\frac{Gmr}{d^2}}$$

Since two stars have the same mass, r=d/2.
The period is

$$T=\frac{2\pi d/2}{v}=\frac{\pi d^{3/2}}{\sqrt{Gm/2}}=7.9\ years$$

Question 12

When earthquake occurs, it triggers P-waves (longitudinal waves) and S-waves (transverse waves) at the same time. The average propagation velocities of P-waves and S-waves in the Earth's crust are known to be 5.5 km/s and 3.2 km/s, respectively. During an earthquake, a seismic monitoring station recorded the arrival times of P-waves and S-waves at 13hr 12min 24s and 13hr 12min 36s. Determine the distance between the station and the center of the earthquake in kilometers.

当地震发生时，P-波(纵波)与 S-波(横波)会同时被触发。已知 P-波与 S-波在地壳中的平均速率分别为 5.5 km/s 和 3.2 km/s。某次地震发生后，观测站记录到 P-波与 S-波到达的时刻分别为 13 时 12 分 24 秒 及 13 时 12 分 36 秒。计算地震中心与观测站的距离，以公里为单位。

Ans: 92

Assume that the earthquake occurs at time t_0, distance R from the station

$$\Delta t=(t_s-t_0)-(t_p-t_0)$$

$$\Delta t=\frac{R}{v_s}-\frac{R}{v_p}$$

$$R=\frac{v_s v_p}{v_p-v_s}\Delta t=\frac{5.5\times3.2}{5.5-3.2}\ 12=91.8\text{ km}$$

Question 13

An insulated beaker with negligible mass contains 0.25 kg of water at 70° C. How many grams of ice at −10 °C must be dropped into the water to make the final equilibrium temperature of the system 30 °C? (Given the fusion heat of ice 334000 J/kg, specific heat of ice and water 2100 J/kg°C ,4187 J/kg°C respectively.)

一质量可以忽略的热绝缘烧杯中装有 0.25 kg，70° C 的水。需要放入多少克，温度在 −10 °C 的冰块，才能使最终的平衡温度达到 30 °C? (取冰的熔化潜热为 334000 J/kg，冰和水的比热分别为 2100 J/kg°C ,4187 J/kg°C。)

Ans: 87

$$m_w c_w \Delta T_w = m_i L_i + m_i c_i \Delta T_i + m_i c_w \Delta T_{iw}$$

$$0.25(4187)(70-30)=m_i(334000+2100\times10+4187\times(30\text{-}0))$$

$$m_i=0.087\ kg$$

Question 14

An ideal Carnot engine is operating between 477 °C and 177 °C with a heat input of 500 J per cycle. If this engine is used to perform a work so that a 50 kg rock is lifted through a height of 10 m, at least how many number of cycles is required for the engine?.

某理想卡诺热机操作在温度 477 °C 与 177 °C 之间，每一次热机循环吸收的热量为 500 J。若使用此热机做功，把 50 kg 的石头提升 10 m 高，热机至少需要经历几次循环？

Ans: 25

$$\frac{W}{Q_H}=1-\frac{T_L}{T_H}$$

$$W=(1-\frac{450}{750})\ 500\ =200\text{ J}$$

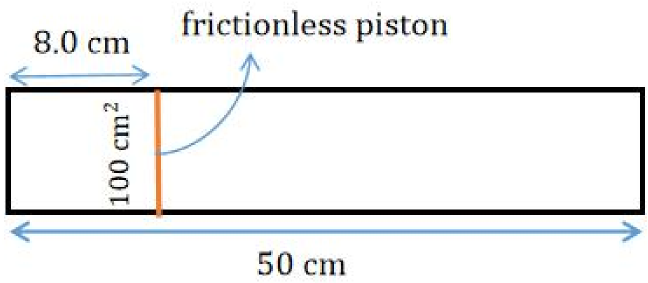
Energy required to lift the rock $E=mgh=50(9.8)10=4900J$

The minimum number of cycles = $E/W=24.5$

Question 15

A cylindrical tube containing an ideal gas is placed on a smooth horizontal surface. The tube has a length of 50 cm and is divided into two compartments by a freely sliding piston with an area of $100\ \mathrm{cm}^2$. The thickness of the piston is negligible. Initially, the pressures in the left and right compartments are both $0.02\ \mathrm{N/cm}^2$, the piston's equilibrium position is 8.0 cm from the left end. When the tube is pushed to the left with a horizontal force of 2.0 N, the piston displaces x cm relative to the tube. Determine the value of x. (Assume the temperature of the gas remains constant.)

一装有理想气体的圆柱管水平置放在光滑平面上。圆柱管的长度为 50 cm，内部被面积为 $100\ \mathrm{cm}^2$，可以自由滑动的活塞分割成两部分。活塞的厚度可以忽略不计。初始时，左右两室的压强均为 $0.02\ \mathrm{N/cm}^2$，活塞的平衡位置在离左端 8.0 cm 处。当圆柱管被 2.0 N 向左的水平力推动时，活塞相对于圆柱管移动了 x 公分。求 x 的数值。(假设气体温度维持不变。)



Ans: 12

Assume the final equilibrium position is x relative to the left end.

$$8.0P_0 = P_L x$$

$$42P_0 = P_R(50-x)$$

When the piston is equilibrium again,

$$P_R A - P_L A = F$$

$$\frac{42P_0}{50-x}A - \frac{8.0P_0}{x}A = F$$

$$42P_0 Ax - 8.0P_0 A(50-x) = F(50-x)x$$

$$2.0x^2 - 800 = 0$$

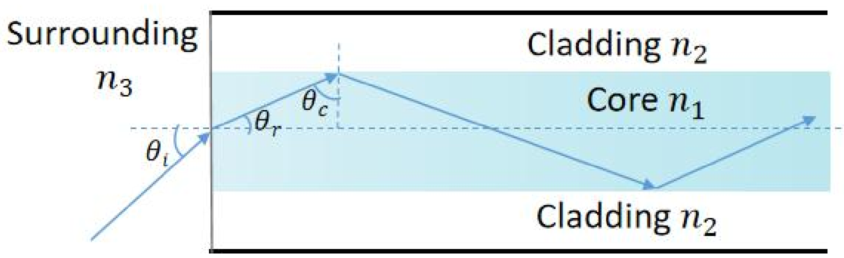
$$x = 20$$

Displacement $(20-8) = 12\ cm$

Question 16

The optical model of an endoscope consists of an optical fiber with a refractive index n_1, covered by a cladding with a refractive index n_2. The end of the fiber is surrounded by an environment with a refractive index n_3. By appropriately selecting the refractive indices, light entering from the end can be transmitted to the other end of the fiber via total internal reflection. Given n_1=1.6, n_2=1.35, and n_3=1, determine the maximum incident angle at the end for which light can be successfully transmitted through the fiber.

内窥镜的光学模型是一根折射率为n_1的光纤，其外围包覆着折射率为n_2的包层。光纤的末端被折射率为n_3的环境围绕着，适当的选取折射率的数值能够确保来自末端的入射光可以在光纤中以全反射的方式传输到另一端。已知n_1=1.6, n_2=1.35, 及 n_3=1，求末端入射光能在光纤中顺利传输的最大入射角度。



Ans: 59

$$\frac{\sin\theta_i}{\sin\theta_r}=\frac{n_1}{n_3},\qquad \frac{\sin 90}{\sin\theta_c}=\frac{n_1}{n_2}$$

Where $\theta_r=90-\theta_c$, θ_i is the maximum incident angle that ensure the total internal reflection in the fibre.

$$\frac{\sin\theta_i}{\sin(90-\theta_c)}=\frac{n_1}{n_3}$$

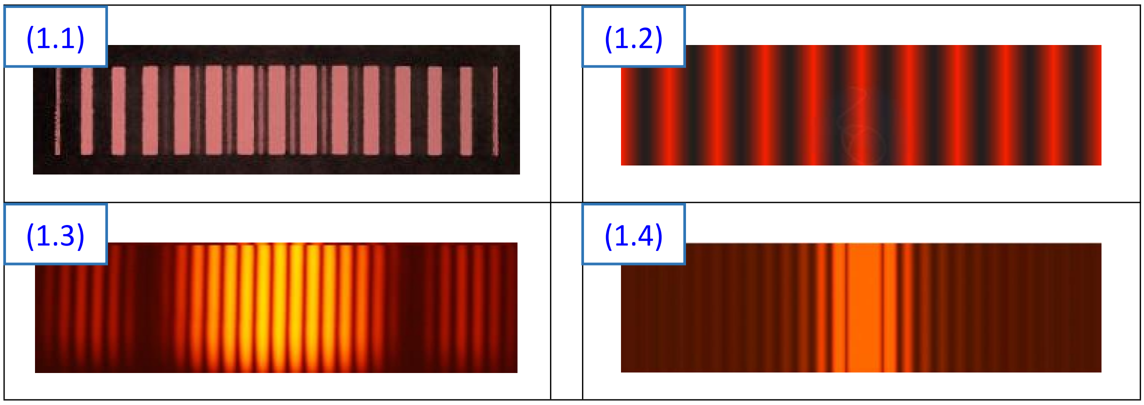
$$\sin\theta_i=n_1\cos\theta_c=n_1\sqrt{1-(\frac{n_2}{n_1})^2}=\sqrt{n_1^2-n_2{}^2}$$

$$\theta_i=59$$

Question 17

In the theoretical analysis of Young's double-slit experiment, we assume that the widths of the two slits are infinitely small. However, the double slits used in the laboratory always have a finite width. Therefore, which of the following sets of fringes would be observed in a laboratory double-slit experiment? Select the number corresponding to the correct fringe pattern.

在对杨氏双缝实验进行理论分析时，我们假设双缝各自的宽度是无限小的。然而实验室中使用的双缝总是具有有限的宽度。因此，以下那一组条纹会是在实验室中观察到的双缝实验条纹？选取正确条纹对应的数值。

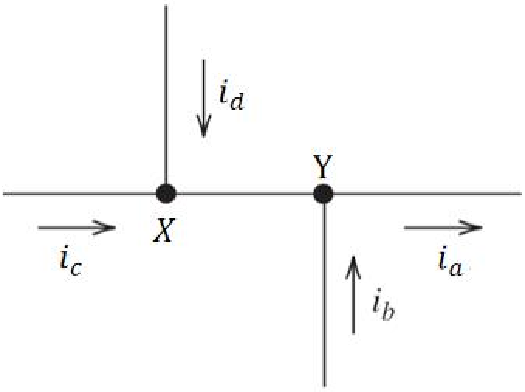


Ans: 1.3

Question 18

Figure below shows part of a circuit. X and Y are two junctions on the circuit. Given that i_a=2.0A, i_c=3.4 A, and i_d=1.2A, what is the value of current i_b in Amperes (A)?.

下图显示某电路的一部分。其中 X 和 Y 为电路上的两个结点。已知i_a=2.0A, i_c=3.4 A, 及 i_d=1.2 A， i_b的数值应为多少？ 以 Ampere (A)为单位。



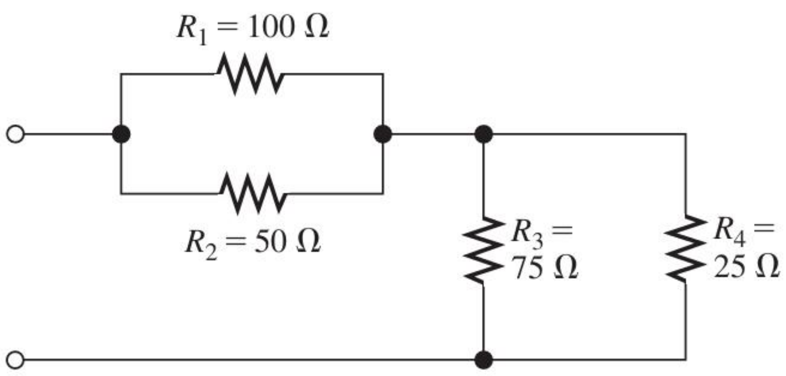
Ans: -2.6

$1.2 + 3.4 + i_b = 2 \rightarrow$ **i_b = -2.6 A**

Question 19

Find the equivalent resistance in the following circuit.

求以下电路的等效电阻，以 Ohms (Ω)为单位。



Ans: 52

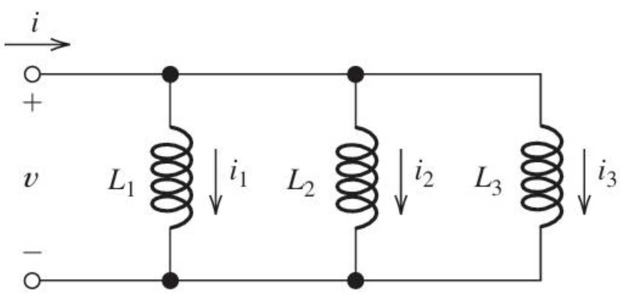
$R_1//R_2 = \frac{100\times50}{150} 33.33\ \Omega$ $R_3//R_4 = \frac{75\times25}{100} 18.75\ \Omega$

$R_{eq} = 33.33 + 18.75 = \mathbf{52.08\ \Omega}$

Question 20

Three inductors are connected in parallel. Given that $L_1 = L_2 = 4.0$ mH and $L_3 = 3.0$ mH, find the equivalent inductance in mH.

三个电感器并联，已知电感 $L_1 = L_2 = 4.0$ mH 及 $L_3 = 3.0$ mH, 求等效电感，以 milliHenry (mH) 为单位。



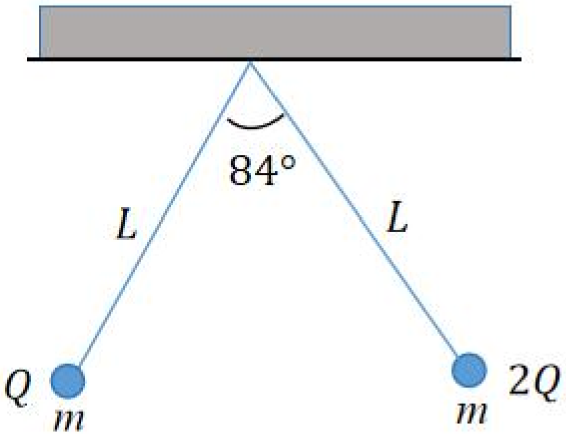
Ans: 1.2

$$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + 1/L_3} = \frac{1}{1/4 + 1/4 + 1/3} = 1.2\ mH$$

Question 21

Two small balls of mass m=1.0 g are suspended at the same point by insulating threads of length L=30 cm. One of the balls carries electric charge Q and the other 2Q. The two balls repel each other and eventually reach equilibrium with an angle 84°. Determine the charge Q in micro Coulombs(μC). (take $\frac{1}{4\pi\varepsilon_0}=9.0\times10^9\ \mathrm{Nm^2/C^2}$)

两质量各为 m=1.0 g 的小球分别以长 L=30 cm 的绝缘线悬挂在同一点。其中一小球带电量 Q，而另一小球带电量 2Q。两小球相互排斥后平衡在 84°张角。求电量 Q 以 micro Coulombs (μC)为单位。(取 $\frac{1}{4\pi\varepsilon_0}=9.0\times10^9\ \mathrm{Nm^2/C^2}$)



Ans: 0.28

$$\tan\theta = K\frac{2Q^2}{mg\,4L^2\sin^2\theta}$$

$$Q^2 = \frac{1}{K}2mgL^2\sin^2\theta\tan\theta$$

$$= \frac{2}{9.0\times10^9}0.001(9.8)0.3^2(\sin42°)^2\tan42°$$

$$Q = 0.28\times10^{-6}\ \mathrm{C}$$

Question 22

A wire 25 cm long lies a long the z-axis and carries a current of 7.4 A in +z-direction. The wire is located in an region with a uniform magnetic field. The components of the magnetic field are given by $(B_x, B_y, B_z) = (-0.23, -0.78, -0.34)$ T.

Find the magnitude of the magnetic force (in Newtons, N) on the wire.

一长 25 cm 的导线平行于 z 轴，带有 7.4A 沿+z 方向的电流。导线处在均匀磁场中，磁场的分量分别为 $(B_x, B_y, B_z) = (-0.23, -0.78, -0.34)$ T。求作用在导线上磁力的大小，以 Newtons(N)为单位。

Ans: 1.5

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$= 7.4 \times 0.25(0,0,1) \times (-0.23, -0.78, -0.34)$$

$$= 1.85(+0.78, -0.23, 0)$$

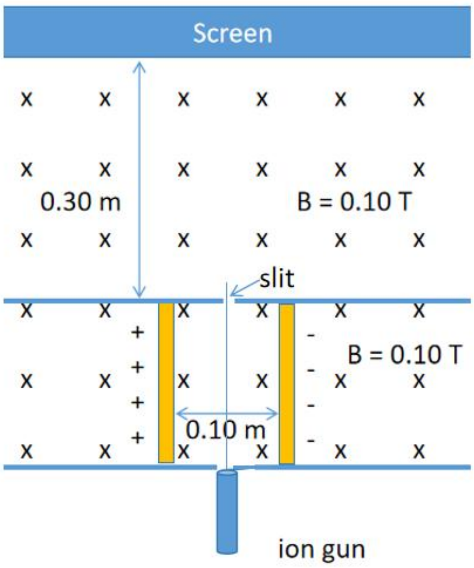
Magnitude

$$F = 1.85\sqrt{0.78^2 + 0.23^2} = 1.5\ N$$

Question 23

An ion gun ejects an unknown ion into a region with a uniform magnetic field of 0.10 T, directed into the plane as shown below. Two parallel plates aparted0.10 m apply a potential difference V to select the ion's speed. When the $V=2.0\times10^{3}$ V, the ions pass through the slit and enter another region with same magnetic field but no parallel plates. These ion beam thereby deflects. At the end, the beam hits the screen located 30 cm from the slit, with a deflected distance of 8.7 cm.. Determine the mass-to-charge ratio of the ion in unit 10^{-3} g/C

一离子枪发射离子束进入均匀磁场强度为 0.10 T 的区域，磁场的方向指入平面内，如图所示。两相距 0.10m 的平行电板以电势差 V 来筛选离子的速度。当 $V=2.0\times10^{3}$ V，离子束能够毫无偏折的垂直通过狭缝进入到另一个具有相同磁场的区域，此区域没有平行电板，离子束因此开始偏折。离子束最后打在离狭缝 30cm 的屏幕上，偏离中心 8.7 cm。求离子的质量电荷比值，以 10^{-3} g/C 为单位。



Ans: 0.28

$$v = V/dB = 2.0\times10^{5}\ m/s$$

$$R = \frac{mv}{qB} = \frac{m}{q}\,2.0\times10^{6}$$

Let s be the deflected distance from center

$$s = R - \sqrt{R^2 - h^2}$$

$$R - s = \sqrt{R^2 - h^2}$$

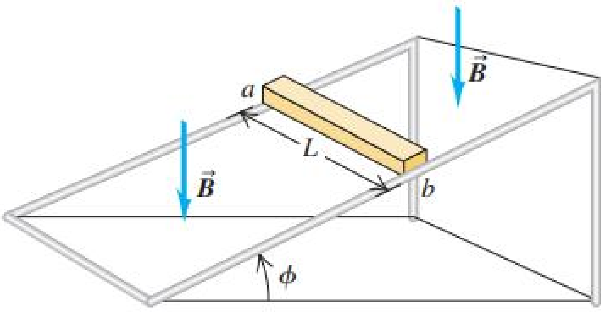
$$R^2 = \left(\frac{h^2 + s^2}{2s}\right)^2$$

$$\frac{m}{q} = \frac{1}{2.0\times10^{6}}\frac{0.3^2 + 0.087^2}{2(0.087)} = 0.28\ \text{mg/C}$$

Question 24

A metal bar with length L=1.0 m, mass m=145 g, and resistance R=8.8 Ω is placed on frictionless metal rails that are inclined at an angle ϕ=30° above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B=0.4 T is directed downward as shown in figure. The bar is released from rest and slides down the rails. Calculate the terminal speed of the bar in unit m/s.

长度 L=1.0 m，质量 m=145 g，及电阻值 R=8.8 Ω的金属棒放置在一金属滑轨上，滑轨与水平面成 ϕ=30°，其电阻可以忽略。施加磁场强度 B=0.4 T 的均匀磁场垂直水平面向下，如图所示。当金属棒由静止开始下滑，计算它的终端速度，以 m/s 为单位。



Ans: 39

$$F=ILB=\frac{\varepsilon}{R}LB$$

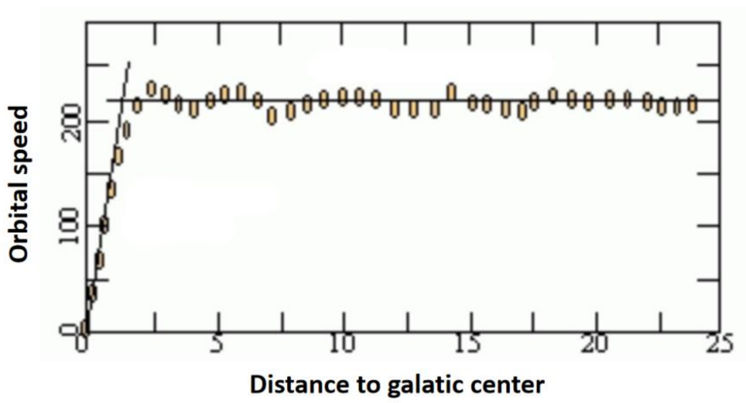
$$mg\sin\theta=\frac{vLB}{R}LB$$

$$v=\frac{Rmg\sin\theta}{L^2B^2}=\frac{8.8(0.145)9.8\sin 30}{1.0^2 0.4^2}=39\ m/s$$

Question 25

Observations of galaxies show that the orbital speed of stars around the galactic center is independent from the distance, for large distances from the center. This behavior deviates from what is expected under Newtonian gravity. Assume that this phenomenon is due to a modified gravitational force of the form $F = G\,m\,M/r^k$, where r is the distance from the galactic center, and k is an unknown exponent, m is the mass of star, M is the total mass within the distance r, and G is gravitational constant. What value of k would result in a constant orbital speed, consistent with observations? **Give the answer to one decimal place.**

星系观测显示，远离星系中心的星体，其轨道速率与离星系中心的距离无关。这一行为偏离了牛顿引力的预测。假设这个现象可以通过修正的引力公式，$F = G\,m\,M/r^k$ 来解释，其中 r 是星体到星系中心的距离，k 是未知的指数。m 是星体质量，M 是在 r 以内的总质量，G 为引力常数。求 k 的值以使远处的星体轨道速率与距离无关。答案准确至一位小数。



Ans: 1.0

The modified gravity is the centrifugal force, G $\frac{m\,M}{r^k} = \frac{m\,v^2}{r}$. From here $v = \sqrt{G\,\frac{r\,M}{r^k}}$. To make this independent of the distance, r has to be cancelled out. Hence **k = 1.0**.

Question 26

One reason the photoelectric effect cannot be explained by classical physics is that, according to classical prediction, it would take much longer than observed to eject an electron. Consider the following scenario. A 2.0-Watt light bulb is placed 1.0 meter away from a potassium plate. Assume an electron in the plate occupies an area modeled as a circle with a radius of 1.0 Å. The energy required to remove the electron from the potassium surface is 2.1 eV. How long would it take, under classical assumptions, for the electron to absorb enough energy to be ejected from the plate? **Give the answer in seconds.**

其中一个古典物理无法解释光电效应的原因是，古典物理预测需要足够长的时间才能弹出一个电子，然而实际观测只要光的频率高于某个阈值，光电子的产生几乎是一瞬间的。考虑这样一个情况，一个 2.0 W 的灯泡放在离金属钾 1.0 m 处。假设一个电子占据的面积大约等于半径为 1.0 Å的圆。要让一个电子自钾金属表面脱离需要的能量为 2.1 eV。从古典物理的角度估算，电子需要多久的时间才能吸收足够的能量脱离金属表面？答案以秒为单位。

Ans: 67

The bulb emits 2.0 J of energy per second uniformly around. The part of this energy that hits the electron is given by the area of the electron cloud ($a = \pi r^2$, with $r = 10^{-10}\ m$) divided by the total area of the sphere with radius R=1.0 m, i.e. the ratio is $\frac{\pi r^2}{4\pi R^2} = 0.25 \times 10^{-20}$. Accordingly, in 1 second the electron absorbs $0.5 \times 10^{-20}\ J = 0.03125\ eV$. Hence to absorb 2.1 eV it takes 2.1/0.03125 = 67.2 **seconds**.

More than 1 minutes! In the experiment we observe the electron to be emitted practically instantaneously.

Question 27

According to thermal physics, the probability of a hydrogen atom occupying an energy level E_n is proportional to the Boltzmann factor, $P_n \propto e^{-\frac{E_n}{kT}}$, where k=8.6 ×10^{-5} eV/K is the Boltzmann constant. When hydrogen gas is in thermal equilibrium with its environment at temperature, T = 304 K, calculate the natural logarithm of the ratio of the probabilities that a hydrogen atom is found in the first excited state (n=2) versus the ground state (n=1), $\ln(P_2/P_1)$. **Round the answer to integer**. (Ionization energy of a hydrogen atom 13. 6 eV.)

根据热物理，氢原子占据能级为E_n的概率与玻尔兹曼因子成正比, $P_n \propto e^{-\frac{E_n}{kT}}$。其中玻尔兹曼常数取值 k=8.6 ×10^{-5} eV/K。当氢气在温度为 T = 304 K 的环境下达到热平衡, 计算 一氢原子处在激发态(n=2)与处在基态(n=1)的概率比值的自然对数, $\ln(P_2/P_1)$。答案以整数表示。(氢原子的游离能为 13. 6 eV。)

Ans: -390

The ratio of probabilities is given by the Boltzmann factor $p_e/p_g = e^{-\frac{E_e - E_g}{kT}}$. From Bohr model $E_e - E_g = \left(-\frac{1}{4}+1\right) 13.6\ eV = 10.2\ eV$. Hence, the logarithm of the ratio is -$\frac{E_e - E_g}{kT}$ = **-390**.

Hence at room temperature practically all hydrogen is in the ground state.

Question 28

In an experiment, a muon is observed to travel 800 m before decaying. The proper lifetime of a muon is given in reference tables as 2.0×10^{-6}s. Find the speed of the muon in the lab frame **in unit** 10^8m/s.

在某个实验中，一个μ粒子在经过 800 m 后发生衰变。已知μ粒子的固有时为 2.0×10^{-6}s，求μ粒子在实验室坐标系中的速率，以 10^8m/s 为单位。

Ans: 2.4

It's quite easy to make an error here. Namely, one could think that the answer is $\frac{800}{2} \times 10^{-6} = 4 \times 10^8\ m/s$, which cannot be the case as this is larger than c, and shows that relativistic effects are important. Indeed, in the laboratory frame the speed of the muon is v = d/t, where $t = \gamma t' = \sqrt{1 - \frac{v^2}{c^2}} / t'$ due to time dilation. Solving for v we find $v = \frac{d\,c}{\sqrt{d^2 + t'^2 c^2}} = 2.4 \times 10^8\ m/s$. This is **0.8 c**.

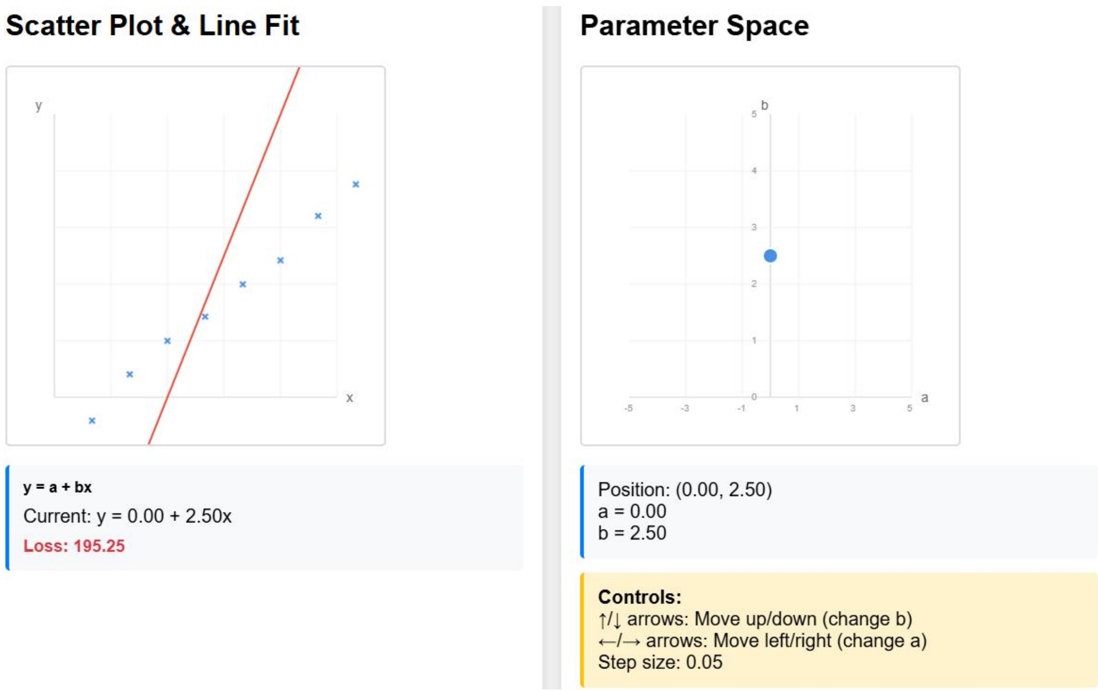
Question 29

In many physics experiments, we use linear regression to fit experimental data. Linear regression is the process of finding the appropriate coefficients **a** and **b** such that the straight-line, y = a+bx best approximates the experimental data. But what does “best approximates” mean? In machine learning, we define a loss function to measure the average difference between the experimental data and the values predicted by the equation. “Best approximating” the data means finding the appropriate values **a** and **b** that minimize the value of this loss function.

Now, imagine you are a machine. Given a set of experimental data, you can “scan” all combinations of **a** and **b** using the arrow keys and get the corresponding loss function value for each combination. Your task is to determine the optimal values for **a** and **b**. Keep to **TWO** decimal place for the final answer.

在许多物理实验中，我们会使用线性回归来拟合实验数据。所谓的线性回归就是寻找适当的系数 **a** 和 **b**，使得直线方程 y = a+bx，最接近实验数据。然而什么是”最接近”？在机器学习里， 我们定义一个损失函数来比较实验数据与方程预测值的平均差异。最接近实验数据的意思就是寻找适当的系数 **a** 和 **b**，使得损失函数的值最小。

假设你就是电脑，给予一组实验数据，你可以通过上下左右键来扫描所有 **a** 和 **b** 的组合并得出相应损失函数的值，请决定最佳的 **a** 和 **b** 的数值。最后答案保留至**小数两位**。



Ans: -1.6, 0.85

Within the resolution, when a = 1.6, b = 0.85, the loss has a minimum value 0.12.

Question 30

The Ruchardt experiment is a classic physics experiment designed to measure the ratio of specific heats of a gas.

$$\gamma = \frac{C_P}{C_V}$$

The basic setup consists of a glass tube filled with gas and sealed by a lightweight piston which can move freely up and down with minimal friction. When the piston is at equilibrium position, its weight is balanced by the pressure difference inside and outside the tube.

$$mg = (P_{eq} - P_0)A$$

Where P_{eq} is the equilibrium pressure in the tube, P_0 is the atmosphere pressure outside the tube, g is the gravitational acceleration, A and m are the area and the mass of the piston, respectively. When the experiment starts, the piston is displaced from its equilibrium position. After released, it will oscillate in approximate simple harmonic motion. If we assume the compression expansion of the gas during the oscillation is adiabatic, we can get the value of γ by measuring the period, T of the oscillation.

$$T^2 = 4\pi^2 \frac{mH}{\gamma A P_{eq}}$$

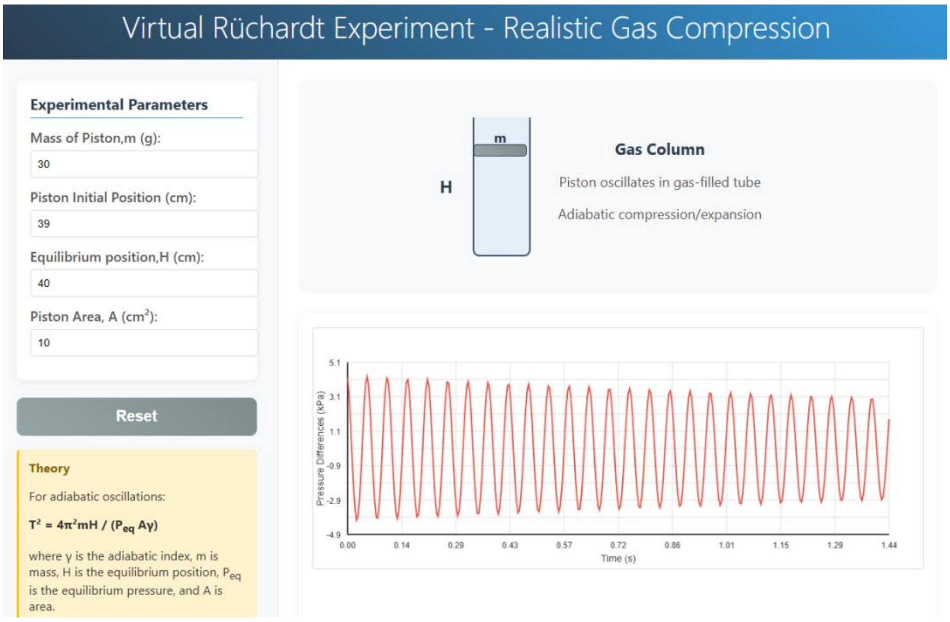
Where H is the equilibrium height of the piston.

Repeat the experiment with the following values of H,

H=35 cm, 30 cm, 25 cm, 20 cm, and 15 cm.

at fixed parameters： $m=50$ g, $A=5.0$ cm^2, $P_0=101000$ N/m^2

Determine the value of γ by measuring the periods of the oscillation. Keep to one decimal place for the final answer.



Ruchardt 实验是一个用来测量气体热容的比值的经典实验。

$$\gamma=\frac{C_P}{C_V}$$

实验的基本装置包含了一个充满气体的玻璃管被可以自由上下活动的活塞密封。当活塞处在平衡位置时，活塞的重量被玻璃管内外的压力差平衡。

$$mg=(P_{eq}-P_0)A$$

其中 P_{eq}为平衡时玻璃管中的压强， P_0为管外大气压强，g 为重力加速度，A 为活塞的面积。实验开始时，活塞被稍微偏离平衡位置，被释放后，活塞会作近似于简谐运动的振动。若在振动过程中，玻璃管内的气体压缩和膨胀过程可看成是绝热过程，则可以通过测量振动周期来获得γ的数值。

$$T^2=4\pi^2\frac{mH}{\gamma AP_{eq}}$$

其中 H 为活塞平衡时的高度。

请用以下 H 的数值重复实验

H=35 cm, 30 cm, 25 cm, 20 cm, and 15 cm.

固定参数 m=50 g, A=5.0 cm^2, P_0=101000 N/m^2。

测量振动周期以求出γ的数值。最后答案保留至小数一位。

Ans: 1.6

H/m	**T/s**	T^2/s^2	$\gamma=4\pi^2\frac{mH}{AP_{eq}T^2}$
0.35	**0.092**	**0.00846**	**1.601**
0.30	**0.085**	**0.0072**	**1.607**
0.25	**0.0775**	**0.006**	**1.611**
0.20	**0.0688**	**0.00473**	**1.635**
0.15	**0.0593**	**0.00352**	**1.649**

Alternatively, plot T^2 vs H and find the slope using linear regression.

Or average the five values of γ.

~~~ End of Questions ~~~