

Prof. Dr. Darren Ong Chung Lee Publishes Proof on Simon's OPUC Hausdorff Dimension Conjecture

Professor Dr. Darren Ong Chung Lee from the Department of Mathematics and Applied Mathematics, along with David Damanik from Rice University and Shuzheng Guo from Ocean University of China recently published a journal article titled "Simon's OPUC Hausdorff dimension conjecture" in *Mathematische Annalen*, which is a Q1 journal in for mathematics research.

The authors prove the OPUC Hausdorff Dimension Conjecture, raised by Barry Simon in 2005. This conjecture is regarding probability measures on a circle. A probability measure represents the likelihood that a particle appears at a specific point of the circle.

Using the theory of orthogonal polynomials on the unit circle (OPUC), this probability measure also gives us a sequence of recurrence coefficients, known as Verblunsky coefficients. There are many connections between the decay rate of these Verblunsky coefficients, and how "smooth" the corresponding probability measures are. One way to measure this smoothness is to calculate the Hausdorff dimension of the probability measure. Simon's OPUC Hausdorff Dimension conjecture gives an explicit formula connecting the decay rate of these coefficients and the Hausdorff dimension of the measure. This formula was conjectured by Barry Simon, and proven in this paper.

This result has many applications in quantum mechanics, for example these Hausdorff dimensions are connected to the spreading rate of quantum random walks. The paper can be found at <https://doi.org/10.1007/s00208-021-02283-7>.

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